

Absence of the Fifth Force Problem in a Model with Spontaneously Broken Dilatation Symmetry

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Abstract

A scale invariant model containing dilaton ϕ and dust (as a model of matter) is studied where the shift symmetry $\phi \rightarrow \phi + \text{const.}$ is spontaneously broken at the classical level due to intrinsic features of the model. The dilaton to matter coupling "constant" f appears to be dependent of the matter density. In normal conditions, i.e. when the matter energy density is many orders of magnitude larger than the dilaton contribution to the dark energy density, f becomes less than the ratio of the "mass of the vacuum" in the volume occupied by the matter to the Planck mass. The model yields this kind of "Archimedes law" without any especial (intended for this) choice of the underlying action and without fine tuning of the parameters. The model not only explains why all attempts to discover a scalar force correction to Newtonian gravity were unsuccessful so far but also predicts that in the near future there is no chance to detect such corrections in the astronomical measurements as well as in the specially designed fifth force experiments on intermediate, short (like millimeter) and even ultrashort (a few nanometer) ranges. This prediction is alternative to predictions of other known models.

Keywords: Fifth force; Spontaneously broken dilatation symmetry; Coupling depending on the matter density.

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I. INTRODUCTION

Possible coupling of the matter to a scalar field can be the origin of a long range force if the mass of the scalar particles is very small. It is well known since the appearance of the Brans-Dicke model[1] that such "fifth" force could affect the results of tests of General Relativity (GR). In more general cases it may entail a violation of the Einstein's equivalence principle. A possible existence of light scalar particles interacting to matter could also give rise to testable consequences in an intermediate or submillimeter or even shorter range depending on the scalar mass. Numerous, many years lasting, specially designed experiments, see for example [2]-[10], have not revealed so far any of possible manifestations of the fifth force. This fact, on each stage of the sequence of experiments, is treated as a new, stronger constraint on the parameters (like coupling constant and mass) with hope that the next generation of experiments will be able to discover a scalar force modifying the Newtonian gravity. This is the essence of the fifth force problem in the "narrow sense"[47].

In this paper we demonstrate that it is quite possible that the fifth force problem in such narrow sense does not exist. Namely we will present a model where the strength of the dilaton to matter coupling measured in experimental attempts to detect a correction to the Newtonian gravity turns out so small that at least near future experiments will not be able to reveal it. On the other hand, if the matter is very diluted then its coupling to the dilaton may be not weak. But the latter is realized under conditions not compatible with the design of the fifth force experiments.

The idea of the existence of a light scalar coupled to matter has a well known theoretical ground, for example in string theory[11] and in models with spontaneously broken dilatation symmetry[12],[13]. The fifth force problem has acquired a special actuality in the last decade when the quintessence[14] and its different modifications, for example coupled quintessence[15], k-essence[16], were recognized as successful models of the dark energy[17]. If the amazing observational fact[18] that the dark energy density is about two times bigger than the (dark) matter density in the present cosmological epoch is not an accidental coincidence but rather is a characteristic feature during long enough period of evolution, then the explanation of this phenomenon suggests that there is an exchange of energy between dark matter and dark energy. A number of models have been constructed with the aim to describe this exchange, see for example[15], [19]-[26] and references therein. In the context

of scalar field models of the dark energy, the availability of this energy exchange implies the existence of a coupling of the scalar field to dark matter. Then immediately the question arises why similar coupling to the visible matter is very strongly suppressed according to the present astronomical data[10]. Thus the resolution of the fifth force problem in its modern treatment should apparently consist of simultaneous explanations, on the ground of a fundamental theory, of both the very strong suppression of the scalar field coupling to the visible matter and the absence of similar suppression of its coupling to the dark matter.

One of the interesting approaches to resolution of the fifth force problem known since 1994 as "the least coupling principle" based on the idea[27] to use non-perturbative string loop effects to explain why the massless dilaton may decouples from matter. In fact it was shown that under certain assumptions about the structure of the (unknown) dilaton coupling functions in the low energy effective action resulting from taking into account the full non-perturbative string loop expansion, the string dilaton is cosmologically attracted toward values where its effective coupling to matter disappears.

The astrophysical effects of the matter density dependence of the dilaton to matter coupling was studied in 1989 in the context of a model with spontaneously broken dilatation symmetry in Ref. [13]. However in this model the effect is too weak to be observed now. Another way to describe the influence of the matter density on the fifth force is used in the Chameleon model[28] formulated in 2004. The key point here is the fact that the scalar field effective potential depends on the local matter density ρ_m if the direct coupling of the scalar field to the metric tensor in the underlying Lagrangian is assumed like in earlier models [29],[30]. Therefore the position of the minimum of the effective potential and the mass of small fluctuations turn out to be ρ_m -dependent. In space regions of "high" matter density such as on the Earth or in other compact objects, the effective mass of the scalar field becomes so big that the scalar field can penetrate only into a thin superficial shell of the compact object. As a result of this, it appears to be possible to realize a situation where in spite of a choice for a scalar to matter coupling of order unity, the violation of the equivalence principle is exponentially suppressed. However, for objects of lower density, the fifth force may be detectable and the corresponding predictions are made.

One should note that the model of Ref. [31] with the matter density dependence of the effective dilaton to matter coupling was constructed in 2001 without any specific conjectures in the underlying action intended to solve the fifth force problem[48]. The resolution of the

fifth force problem appears as a result which reads: 1) The local effective Yukawa coupling of the dilaton to fermions in normal laboratory conditions equals practically zero automatically, without any fine tuning of the parameters. The term normal laboratory conditions means that the local fermion energy density is many orders of magnitude larger than the dilaton contribution to the dark energy density. 2) Under the same conditions, the Einstein's GR is reproduced.

One of the main ingredients of the model[31] consists in the realization of the idea [33] that the fifth force problem might be resolved if the theory would possess the approximate global shift symmetry of the scalar field

$$\phi \rightarrow \phi + \text{const.} \quad (1)$$

In the model [31],[32], the global shift symmetry (1) is spontaneously broken in such a way that the effective potential depends on ϕ *only via* $M^4 e^{-2\alpha\phi/M_p}$ where M is an integration constant of the dimensionality of mass that appears as a result of the spontaneous breakdown of the shift symmetry (1). Here $\alpha > 0$ is a parameter of the order of unity and $M_p = (8\pi G)^{-1/2}$. This is a way the model [31],[32] avoids the problem with realization of the global shift symmetry (1) in the context of quintessence type models where the potential is not invariant under the shift of ϕ . The model with such features was constructed in the framework of the Two Measures Field Theory (TMT) [34]-[41].

In the present paper we show that the main results concerning the decoupling and the restoration of the Einstein's GR in the model [31],[32] for fermions (which is rather complicated), remain also true in a macroscopic description of matter (which is significantly simpler). This should make more clear the way of resolution of the fifth force problem in scale invariant TMT models. Our underlying model involves the coupling of the dilaton ϕ to dust in such a form that Lagrangians are quite usual, without any exotic term, and the action is invariant under scale transformations accompanied by a corresponding shift (1) of the dilaton. After spontaneous symmetry breaking (SSB), the effective picture in the Einstein frame differs in general very much from the Einstein's GR. But if the local matter density is many orders of magnitude larger than the vacuum energy density then Einstein's GR is reproduced, and the dilaton to matter coupling practically disappears without fine tuning of the parameters.

II. BASIS OF TWO MEASURES FIELD THEORY AND FORMULATION OF THE SCALE INVARIANT MODEL

A. Main ideas of the Two Measures Field Theory

TMT is a generally coordinate invariant theory where *all the difference from the standard field theory in curved space-time consists only of the following three additional assumptions:*

1. The first assumption is the hypothesis that the effective action at the energies below the Planck scale has to be of the form[34]-[41]

$$S = \int L_1 \Phi d^4x + \int L_2 \sqrt{-g} d^4x \quad (2)$$

including two Lagrangians L_1 and L_2 and two measures of integration $\sqrt{-g}$ and Φ . One is the usual measure of integration $\sqrt{-g}$ in the 4-dimensional space-time manifold equipped with the metric $g_{\mu\nu}$. Another is the new measure of integration Φ in the same 4-dimensional space-time manifold. The measure Φ being a scalar density and a total derivative may be defined for example[49] by means of four scalar fields φ_a ($a = 1, 2, 3, 4$)

$$\Phi = \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\mu \varphi_a \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d. \quad (3)$$

To provide parity conservation one can choose for example one of φ_a 's to be a pseudoscalar.

2. Generically it is allowed that L_1 and L_2 will be functions of all matter fields, the dilaton field, the metric, the connection but not of the "measure fields" φ_a . In such a case, i.e. when the measure fields enter in the theory only via the measure Φ , the action (2) possesses an infinite dimensional symmetry $\varphi_a \rightarrow \varphi_a + f_a(L_1)$, where $f_a(L_1)$ are arbitrary functions of L_1 (see details in Ref. [35]). One can hope that this symmetry should prevent emergence of a measure fields dependence in L_1 and L_2 after quantum effects are taken into account.

3. Important feature of TMT that is responsible for many interesting and desirable results of the field theory models studied so far[31],[32],[34]-[41] consists of the assumption that all fields, including also metric, connection and the *measure fields* φ_a are independent dynamical variables. All the relations between them are results of equations of motion. In particular, the independence of the metric and the connection means that we proceed in

the first order formalism and the relation between connection and metric is not a priori according to Riemannian geometry.

We want to stress again that except for the listed three assumptions we do not make any changes as compared with principles of the standard field theory in curved space-time. In other words, all the freedom in constructing different models in the framework of TMT consists of the choice of the concrete matter content and the Lagrangians L_1 and L_2 that is quite similar to the standard field theory.

Since Φ is a total derivative, a shift of L_1 by a constant, $L_1 \rightarrow L_1 + const$, has no effect on the equations of motion. Similar shift of L_2 would lead to the change of the constant part of the Lagrangian coupled to the volume element $\sqrt{-g}d^4x$. In the standard GR, this constant term is the cosmological constant. However in TMT the relation between the constant term of L_2 and the physical cosmological constant is very non trivial and this makes possible[35],[31],[32],[41] to resolve the cosmological constant problem[50].

Varying the measure fields φ_a , we obtain

$$B_a^\mu \partial_\mu L_1 = 0 \quad \text{where} \quad B_a^\mu = \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d. \quad (4)$$

Since $Det(B_a^\mu) = \frac{4^{-4}}{4!} \Phi^3$ it follows that if $\Phi \neq 0$,

$$L_1 = sM^4 = const \quad (5)$$

where $s = \pm 1$ and M is a constant of integration with the dimension of mass. In what follows we make the choice $s = 1$.

One should notice *the very important differences of TMT from scalar-tensor theories with nonminimal coupling*:

- a) In general, the Lagrangian density L_1 (coupled to the measure Φ) may contain not only the scalar curvature term (or more general gravity term) but also all possible matter fields terms. This means that TMT modifies in general both the gravitational sector and the matter sector;
- b) If the field Φ were the fundamental (non composite) one then instead of (5), the variation of Φ would result in the equation $L_1 = 0$ and therefore the dimensionfull integration constant M^4 would not appear in the theory.

Applying the Palatini formalism in TMT one can show (see for example [35]) that in addition to the usual Christoffel coefficients, the resulting relation between metric and con-

nection includes also the gradient of the ratio of the two measures

$$\zeta \equiv \frac{\Phi}{\sqrt{-g}} \quad (6)$$

which is a scalar field. This means that with the set of variables used in the underlying action (2) (and in particular with the metric $g_{\mu\nu}$) the space-time is not Riemannian. The gravity and matter field equations obtained by means of the first order formalism contain both ζ and its gradient. It turns out that at least at the classical level, the measure fields φ_a affect the theory only through the scalar field ζ .

Variation with respect to the metric yields as usual the gravitational equations. But in addition, if L_1 involves a scalar curvature term then Eq.(5) provides us with an additional gravitational type equation, independent of the former. Taking trace of the gravitational equations and excluding the scalar curvature from these independent equations we obtain a consistency condition having the form of a constraint which determines $\zeta(x)$ as a function of matter fields. It is very important that neither Newton constant nor curvature appear in this constraint which means that the *geometrical scalar field $\zeta(x)$ is determined by other fields configuration* locally and straightforward (that is without gravitational interaction).

By an appropriate change of the dynamical variables which includes a redefinition of the metric, one can formulate the theory in a Riemannian space-time. The corresponding frame we call "the Einstein frame". The big advantage of TMT is that in a very wide class of models, *the gravity and all matter fields equations of motion take canonical GR form in the Einstein frame*. All the novelty of TMT in the Einstein frame as compared with the standard GR is revealed only in an unusual structure of the scalar fields effective potential, masses of particles and their interactions with scalar fields as well as in the unusual structure of matter contributions to the energy-momentum tensor: all these quantities appear to be ζ dependent. This is why the scalar field $\zeta(x)$ determined by the constraint as a function of matter fields, has a key role in dynamics of TMT models. Note that if we were to assume that for some reasons the gravity effects are negligible and choose to work in the Minkowski space-time from the very beginning, then we would lose the constraint, and the result would be very much different from the one obtained according to the prescriptions of TMT with taking the flat space-time limit at the end. This means that the gravity in TMT plays the more essential role than in the usual (i.e. only with the measure of integration $\sqrt{-g}$) field theory in curved space-time.

B. Scale invariant model

In the original frame (where the metric is $g_{\mu\nu}$), a matter content of our TMT model represented in the form of the action (2), is a dust and a scalar field (dilaton). The dilaton ϕ allows to realize a spontaneously broken global scale invariance[36],[37],[31],[32] and together with this it can govern the evolution of the universe on different stages: in the early universe ϕ plays the role of inflaton and in the late time universe it is transformed into a part of the dark energy (for details see Refs. [31],[32],[41]). We postulate that the theory is invariant under the global scale transformations:

$$g_{\mu\nu} \rightarrow e^\theta g_{\mu\nu}, \quad \Gamma_{\alpha\beta}^\mu \rightarrow \Gamma_{\alpha\beta}^\mu, \quad \phi \rightarrow \phi - \frac{M_p}{\alpha}\theta, \quad \varphi_a \rightarrow l_{ab}\varphi_b \quad (7)$$

where $\det(l_{ab}) = e^{2\theta}$ and $\theta = \text{const.}$ Keeping the general structure (2), it is convenient to represent the action in the following form:

$$\begin{aligned} S &= S_g + S_\phi + S_m \\ S_g &= -\frac{1}{\kappa} \int (\Phi + b_g \sqrt{-g}) R(\Gamma, g) e^{\alpha\phi/M_p} d^4x; \\ S_\phi &= \int e^{\alpha\phi/M_p} \left[(\Phi + b_\phi \sqrt{-g}) \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - (\Phi V_1 + \sqrt{-g} V_2) e^{\alpha\phi/M_p} \right] d^4x; \\ S_m &= \int (\Phi + b_m \sqrt{-g}) L_m d^4x, \end{aligned} \quad (8)$$

where

$$R(\Gamma, g) = g^{\mu\nu} (\Gamma_{\mu\nu,\lambda}^\lambda - \Gamma_{\mu\lambda,\nu}^\lambda + \Gamma_{\alpha\lambda}^\lambda \Gamma_{\mu\nu}^\alpha - \Gamma_{\alpha\nu}^\lambda \Gamma_{\mu\lambda}^\alpha)$$

and the Lagrangian for the matter, as collection of particles, which provides the scale invariance of S_m reads

$$L_m = -m \sum_i \int e^{\frac{1}{2}\alpha\phi/M_p} \sqrt{g_{\alpha\beta} \frac{dx_i^\alpha}{d\lambda} \frac{dx_i^\beta}{d\lambda}} \frac{\delta^4(x - x_i(\lambda))}{\sqrt{-g}} d\lambda \quad (9)$$

where λ is an arbitrary parameter. For simplicity we consider the collection of the particles with the same mass parameter m . We assume in addition that $x_i(\lambda)$ do not participate in the scale transformations (7).

In the action (8) there are two types of the gravitational terms and of the "kinetic-like terms" which respect the scale invariance : the terms of the one type coupled to the

measure Φ and those of the other type coupled to the measure $\sqrt{-g}$. Using the freedom in normalization of the measure fields φ_a we set the coupling constant of the scalar curvature to the measure Φ to be $-\frac{1}{\kappa}$. Normalizing all the fields such that their couplings to the measure Φ have no additional factors, we are not able in general to provide the same in terms describing the appropriate couplings to the measure $\sqrt{-g}$. This fact explains the need to introduce the dimensionless real parameters b_g , b_ϕ and b_m . We will only assume that they are positive, have the same or very close orders of magnitude

$$b_g \sim b_\phi \sim b_m \quad (10)$$

and besides $b_m > b_g$. The real positive parameter α is assumed to be of the order of unity. As usual $\kappa = 16\pi G$ and we use $M_p = (8\pi G)^{-1/2}$.

One should also point out the possibility of introducing two different pre-potentials which are exponential functions of the dilaton ϕ coupled to the measures Φ and $\sqrt{-g}$ with factors V_1 and V_2 . Such ϕ -dependence provides the scale symmetry (7). We will see below how the dilaton effective potential is generated as the result of SSB of the scale invariance and the transformation to the Einstein frame.

According to the general prescriptions of TMT, we have to start from studying the self-consistent system of gravity (metric $g_{\mu\nu}$ and connection $\Gamma_{\alpha\beta}^\mu$), the measure Φ degrees of freedom φ_a , the dilaton field ϕ and the matter particles coordinates $x_i^\alpha(\lambda)$, proceeding in the first order formalism.

For the purpose of this paper we restrict ourselves to a zero temperature gas of particles, i.e. we will assume that $d\vec{x}_i/d\lambda \equiv 0$ for all particles. It is convenient to proceed in the frame where $g_{0l} = 0$, $l = 1, 2, 3$. Then the particle density is defined by

$$n(\vec{x}) = \sum_i \frac{1}{\sqrt{-g_{(3)}}} \delta^{(3)}(\vec{x} - \vec{x}_i(\lambda)) \quad (11)$$

where $g_{(3)} = \det(g_{kl})$ and

$$S_m = -m \int d^4x (\Phi + b_m \sqrt{-g}) n(\vec{x}) e^{\frac{1}{2}\alpha\phi/M_p} \quad (12)$$

Following the procedure described in the previous subsection we have to write down all equations of motion, find the consistency condition (the constraint which determines ζ -field as a function of other fields and matter) and make a transformation to the Einstein frame.

We will skip most of the intermediate results and in the next subsection present the resulting equations in the Einstein frame. Nevertheless two exclusions we have to make here.

The first one concerns the important effect observable when varying S_m with respect to $g^{\mu\nu}$:

$$\frac{\delta S_m}{\delta g^{00}} = \frac{b_m}{2} \sqrt{-g} m n(\vec{x}) e^{\frac{1}{2}\alpha\phi/M_p} g_{00}, \quad (13)$$

$$\frac{\delta S_m}{\delta g^{kl}} = -\frac{1}{2} \Phi m n(\vec{x}) e^{\frac{1}{2}\alpha\phi/M_p} g_{kl}. \quad (14)$$

The latter equation shows that due to the measure Φ , *the zero temperature gas generically possesses a pressure*. As we will see this pressure disappears automatically together with the fifth force as the matter energy density is many orders of magnitude larger than the dark energy density, which is evidently true in all physical phenomena tested experimentally.

The second one is the notion concerning the role of Eq. (5) resulting from variation of the measure fields φ_a . With the action (8), where the Lagrangian L_1 is the sum of terms coupled to the measure Φ , Eq. (5) describes a *spontaneous breakdown of the global scale symmetry* (7).

III. EQUATIONS OF MOTION IN THE EINSTEIN FRAME.

It turns out that when working with the new metric (ϕ remains the same)

$$\tilde{g}_{\mu\nu} = e^{\alpha\phi/M_p} (\zeta + b_g) g_{\mu\nu}, \quad (15)$$

which we call the Einstein frame, the connection becomes Riemannian. Since $\tilde{g}_{\mu\nu}$ is invariant under the scale transformations (7), spontaneous breaking of the scale symmetry (by means of Eq.(5)) is reduced in the Einstein frame to the *spontaneous breakdown of the shift symmetry* (1). Notice that the Goldstone theorem generically is not applicable in this kind of models[37].

The transformation (15) causes the transformation of the particle density

$$\tilde{n}(\vec{x}) = (\zeta + b_g)^{-3/2} e^{-\frac{3}{2}\alpha\phi/M_p} n(\vec{x}) \quad (16)$$

After the change of variables to the Einstein frame (15) and some simple algebra, the gravitational equations take the standard GR form

$$G_{\mu\nu}(\tilde{g}_{\alpha\beta}) = \frac{\kappa}{2} T_{\mu\nu}^{eff} \quad (17)$$

where $G_{\mu\nu}(\tilde{g}_{\alpha\beta})$ is the Einstein tensor in the Riemannian space-time with the metric $\tilde{g}_{\mu\nu}$. The components of the effective energy-momentum tensor are as follows

$$T_{00}^{eff} = \frac{\zeta + b_\phi}{\zeta + b_g} \left(\dot{\phi}^2 - \tilde{g}_{00} X \right) + \tilde{g}_{00} \left[V_{eff}(\phi; \zeta, M) - \frac{\delta \cdot b_g}{\zeta + b_g} X + \frac{3\zeta + b_m + 2b_g}{2\sqrt{\zeta + b_g}} m \tilde{n} \right] \quad (18)$$

$$T_{ij}^{eff} = \frac{\zeta + b_\phi}{\zeta + b_g} (\phi_{,k} \phi_{,l} - \tilde{g}_{kl} X) + \tilde{g}_{kl} \left[V_{eff}(\phi; \zeta, M) - \frac{\delta \cdot b_g}{\zeta + b_g} X + \frac{\zeta - b_m + 2b_g}{2\sqrt{\zeta + b_g}} m \tilde{n} \right] \quad (19)$$

Here the following notations have been used:

$$X \equiv \frac{1}{2} \tilde{g}^{\alpha\beta} \phi_{,\alpha} \phi_{,\beta} \quad and \quad \delta = \frac{b_g - b_\phi}{b_g} \quad (20)$$

and the function $V_{eff}(\phi; \zeta)$ is defined by

$$V_{eff}(\phi; \zeta) = \frac{b_g [M^4 e^{-2\alpha\phi/M_p} + V_1] - V_2}{(\zeta + b_g)^2} \quad (21)$$

The dilaton ϕ field equation in the Einstein frame is as follows

$$\begin{aligned} & \frac{1}{\sqrt{-\tilde{g}}} \partial_\mu \left[\frac{\zeta + b_\phi}{\zeta + b_g} \sqrt{-\tilde{g}} \tilde{g}^{\mu\nu} \partial_\nu \phi \right] \\ & - \frac{\alpha}{M_p} \frac{(\zeta + b_g) M^4 e^{-2\alpha\phi/M_p} - (\zeta - b_g) V_1 - 2V_2 - \delta b_g (\zeta + b_g) X}{(\zeta + b_g)^2} \\ & = \frac{\alpha}{M_p} \frac{\zeta - b_m + 2b_g}{2\sqrt{\zeta + b_g}} m \tilde{n} \end{aligned} \quad (22)$$

In the above equations, the scalar field ζ is determined as a function $\zeta(\phi, X, \tilde{n})$ by means of the following constraint (origin of which has been discussed in Sec.2.1):

$$\frac{(b_g - \zeta) (M^4 e^{-2\alpha\phi/M_p} + V_1) - 2V_2}{(\zeta + b_g)^2} - \frac{\delta \cdot b_g X}{\zeta + b_g} = \frac{\zeta - b_m + 2b_g}{2\sqrt{\zeta + b_g}} m \tilde{n} \quad (23)$$

Applying the constraint (23) to Eq.(22) one can reduce the latter to the form

$$\frac{1}{\sqrt{-\tilde{g}}} \partial_\mu \left[\frac{\zeta + b_\phi}{\zeta + b_g} \sqrt{-\tilde{g}} \tilde{g}^{\mu\nu} \partial_\nu \phi \right] - \frac{2\alpha\zeta}{(\zeta + b_g)^2 M_p} M^4 e^{-2\alpha\phi/M_p} = 0, \quad (24)$$

where ζ is a solution of the constraint (23).

One should point out two very important features of the model. First, the ϕ dependence in all the equations of motion (including the constraint) emerges only in the form $M^4 e^{-2\alpha\phi/M_p}$ where M is the integration constant, i.e. due to the spontaneous breakdown of the scale symmetry (7) (or the shift symmetry (1) in the Einstein frame). Second, the constraint (23) is the fifth degree algebraic equation with respect to $\sqrt{\zeta + b_g}$ and therefore generically ζ is a complicated function of ϕ , X and \tilde{n} . Hence generically each of ζ dependent terms in Eqs. (18)-(22) and (24) describe very nontrivial coupling of the dilaton to the matter.

IV. DARK ENERGY IN THE ABSENCE OF MATTER

It is worth to start the investigation of the features of our model from the simplest case when the particle density of the dust is zero: $\tilde{n}(x) \equiv 0$. Then the dilaton ϕ is the only matter which in the early universe plays the role of the inflaton while in the late universe it is the dark energy. The appropriate model in the context of cosmological solutions has been studied in detail in Ref. [41]. Here we present only some of the equations we will need for the purposes of this paper and a list of the main results.

In the absence of the matter particles, the scalar $\zeta = \zeta(\phi, X)$ can be easily found from the constraint (23):

$$\zeta^{(\tilde{n}=0)} = b_g - 2 \frac{V_2 + \delta \cdot b_g^2 X}{M^4 e^{-2\alpha\phi/M_p} + V_1 + \delta \cdot b_g X} \quad (25)$$

In the spatially homogeneous case $X \geq 0$. Then the effective energy-momentum tensor can be represented in a form of that of a perfect fluid

$$T_{\mu\nu}^{eff} = (\rho + p)u_\mu u_\nu - p\tilde{g}_{\mu\nu}, \quad where \quad u_\mu = \frac{\phi_{,\mu}}{(2X)^{1/2}} \quad (26)$$

with the following energy and pressure densities obtained after inserting (25) into the components of the energy-momentum tensor (18), (19) where now $\tilde{n}(x) \equiv 0$

$$\begin{aligned} \rho(\phi, X; M) &\equiv \rho^{(\tilde{n}=0)} \\ &= X + \frac{(M^4 e^{-2\alpha\phi/M_p} + V_1)^2 - 2\delta b_g (M^4 e^{-2\alpha\phi/M_p} + V_1)X - 3\delta^2 b_g^2 X^2}{4[b_g(M^4 e^{-2\alpha\phi/M_p} + V_1) - V_2]}, \end{aligned} \quad (27)$$

$$p(\phi, X; M) \equiv p^{(\tilde{n}=0)} = X - \frac{(M^4 e^{-2\alpha\phi/M_p} + V_1 + \delta b_g X)^2}{4[b_g(M^4 e^{-2\alpha\phi/M_p} + V_1) - V_2]}. \quad (28)$$

Substitution of (25) into Eq. (24) yields the ϕ -equation with very interesting dynamics. The appearance of the nonlinear X dependence in spite of the absence of such nonlinearity in

the underlying action means that our model represents an explicit example of k -essence[16] resulting from first principles. The effective k -essence action has the form

$$S_{eff} = \int \sqrt{-\tilde{g}} d^4x \left[-\frac{1}{\kappa} R(\tilde{g}) + p(\phi, X; M) \right], \quad (29)$$

where $p(\phi, X; M)$ is given by Eq.(28).

In the context of spatially flat FRW cosmology, in the absence of the matter particles (i.e $\tilde{n}(x) \equiv 0$), the TMT model under consideration[41] exhibits a number of interesting outputs depending of the choice of regions in the parameter space (but without fine tuning):

a) *Absence of initial singularity of the curvature while its time derivative is singular.* This is a sort of "sudden" singularities studied by Barrow on purely kinematic grounds[44].

b) Power law inflation in the subsequent stage of evolution. Depending on the region in the parameter space the inflation ends with a *graceful exit* either into the state with zero cosmological constant (CC) or into the state driven by both a small CC and the field ϕ with a quintessence-like potential.

c) Possibility of *resolution of the old CC problem.* From the point of view of TMT, it becomes clear why the old CC problem cannot be solved (without fine tuning) in conventional field theories.

d) TMT enables two ways for achieving small CC without fine tuning of dimensionful parameters: either by a *seesaw* type mechanism or due to a *correspondence principle* between TMT and conventional field theories (i.e theories with only the measure of integration $\sqrt{-g}$ in the action).

e) There is a wide range of the parameters where the dynamics of the scalar field ϕ , playing the role of the dark energy in the late universe, allows crossing the phantom divide, i.e. the equation-of-state $w = p/\rho$ may be $w < -1$ and w asymptotically (as $t \rightarrow \infty$) approaches -1 from below. One can show that in the original frame used in the underlying action (8), this regime corresponds to the negative sign of the measure of integration $\Phi + b_\phi \sqrt{-g}$ of the dilaton ϕ kinetic term[51]. This dynamical effect which emerges here instead of putting the wrong sign kinetic term by hand in the phantom model[46], will be discussed in detail in another paper.

Taking into account that in the late time universe the X -contribution to $\rho^{(\tilde{n}=0)}$ approaches zero, one can see that the dark energy density is positive for any ϕ provided

$$b_g V_1 \geq V_2 \quad (30)$$

Then it follows from (25) that

$$|\zeta^{(\tilde{n}=0)}| \sim b_g. \quad (31)$$

This will be useful in the next section.

V. NORMAL CONDITIONS: REPRODUCING EINSTEIN'S GR AND ABSENCE OF THE FIFTH FORCE PROBLEM

One should now pay attention to the interesting result that the explicit \tilde{n} dependence involving **the same form of ζ dependence**

$$\frac{\zeta - b_m + 2b_g}{2\sqrt{\zeta + b_g}} m \tilde{n} \quad (32)$$

appears simultaneously[52] in the dust contribution to the pressure (through the last term in Eq. (19)), in the effective dilaton to dust coupling (in the r.h.s. of Eq. (22)) and in the r.h.s. of the constraint (23).

Let us analyze consequences of this wonderful coincidence in the case when the matter energy density (modeled by dust) is much larger than the dilaton contribution to the dark energy density in the space region occupied by this matter. Evidently this is the condition under which all tests of Einstein's GR, including the question of the fifth force, are fulfilled. Therefore if this condition is satisfied we will say that the matter is in **normal conditions**. The existence of the fifth force turns into a problem just in normal conditions. The opposite situation may be realized (see Refs. [31],[32]) if the matter is diluted up to a magnitude of the macroscopic energy density comparable with the dilaton contribution to the dark energy density. In this case we say that the matter is in the state of cosmo-low energy physics (**CLEP**). It is evident that the fifth force acting on the matter in the CLEP state cannot be detected now and in the near future, and therefore does not appear to be a problem. But effects of the CLEP may be important in cosmology, see Ref. [32].

The last terms in eqs. (18) and (19), being the matter contributions to the energy density (ρ_m) and the pressure ($-p_m$) respectively, generally speaking have the same order of magnitude. But if the dust is in the normal conditions there is a possibility to provide the desirable feature of the dust in GR: it must be pressureless. This is realized provided that in normal conditions (n.c.) the following equality holds with extremely high accuracy:

$$\zeta^{(n.c.)} \approx b_m - 2b_g \quad (33)$$

Remind that we have assumed $b_m > b_g$. Then $\zeta^{(n.c.)} + b_g > 0$, and the transformation (15) and the subsequent equations in the Einstein frame are well defined. Inserting (33) in the last term of Eq. (18) we obtain the effective dust energy density in normal conditions

$$\rho_m^{(n.c.)} = 2\sqrt{b_m - b_g} m\tilde{n} \quad (34)$$

Substitution of (33) into the rest of the terms of the components of the energy-momentum tensor (18) and (19) gives the dilaton contribution to the energy density and pressure of the dark energy which have the orders of magnitude close to those in the absence of matter case, Eqs. (27) and (28). The latter statement may be easily checked by using Eqs. (25), (31), (33) and (10).

Note that Eq. (33) is not just a choice to provide zero dust contribution to the pressure. In fact it is the result of analyzing the equations of motion together with the constraint (23). In the Appendix we present the detailed analysis yielding this result. But in this section we have started from the use of this result in order to make the physical meaning more distinct.

Taking into account our assumption (10) and Eq. (31) we infer that $\zeta^{(n.c.)}$ and $\zeta^{(\tilde{n}=0)}$ (in the absence of matter case, Eq. (25)) have close orders of magnitudes. Then it is easy to see (making use the inequality (30)) that the l.h.s. of the constraint (23), as $\zeta = \zeta^{(n.c.)}$, has the order of magnitude close to that of the dark energy density $\rho^{(\tilde{n}=0)}$ in the absence of matter case discussed in Sec. 4. Thus in the case under consideration, *the constraint (23) describes a balance between the pressure of the dust in normal conditions on the one hand and the vacuum energy density on the other hand*. This balance is realized due to the condition (33).

Besides reproducing Einstein equations when the scalar field and dust (in normal conditions) are sources of the gravity, *the condition (33) automatically provides a practical disappearance of the effective dilaton to matter coupling*. Indeed, inserting (33) into the ϕ -equation written in the form (24) and into $V_{eff}(\phi; \zeta)$, Eq. (21), one can immediately see that only the force of the strength of the dark energy selfinteraction is present in this case. Note that this force is a total force involving both the selfinteraction of the dilaton and its interaction with dust in normal conditions. Furthermore, in this way one can see explicitly that due to the factor $M^4 e^{-2\alpha\phi/M_p}$, this total force may obtain an additional, exponential dumping since in the cosmological context shortly discussed in Sec. 4 (see details in Ref. ([41])) a scenario, where in the late time universe $\phi \gg M_p$, seems to be most appealing.

Another way to see the absence of the fifth force problem in the normal conditions is to

look at the ϕ -equation in the form (22) and estimate the Yukawa type coupling constant in the r.h.s. of this equation. In fact, using the constraint (23) and representing the particle density in the form $\tilde{n} \approx N/v$ where N is the number of particles in a volume v , one can make the following estimation for the effective dilaton to matter coupling "constant" f defined by the Yukawa type interaction term $f\tilde{n}\phi$ (if we were to invent an effective action whose variation with respect to ϕ would result in Eq. (22)):

$$f \equiv \alpha \frac{m}{M_p} \frac{\zeta - b_m + 2b_g}{2\sqrt{\zeta + b_g}} \approx \alpha \frac{m}{M_p} \frac{\zeta - b_m + 2b_g}{2\sqrt{b_m - b_g}} \sim \frac{\alpha}{M_p} \frac{\rho_{vac}}{\tilde{n}} \approx \alpha \frac{\rho_{vac} v}{NM_p} \quad (35)$$

Thus we conclude that *the effective coupling "constant" of the dilaton to matter in the normal conditions is of the order of the ratio of the "mass of the vacuum" in the volume occupied by the matter to the Planck mass taken N times*. In some sense this result resembles the *Archimedes law*. At the same time Eq. (35) gives us an estimation of the exactness of the condition (33).

VI. DISCUSSION AND CONCLUSION

In the present paper, the idea to construct a model with spontaneously broken dilatation invariance where the dilaton dependence in all equations of motion results only from the SSB of the shift symmetry (1), is implemented from first principles in the framework of TMT.

Although the dust model studied in this paper is a very crude model of matter, it is quite sufficient for studying the fifth force problem. In fact, all experiments which search for the fifth force deal with macroscopic bodies which, in the zeroth order approximation, can be regarded as collections of noninteracting, point-like motionless particles with very high particle number density $\tilde{n}(x)$.

Generically the model studied in the present paper is different from Einstein's GR. For example it allows the long range scalar force and a non-zero pressure of the cold dust. However the magnitude of the particle number density turns out to be the very important factor influencing the strength of the dilaton to matter coupling. This happens due to the constraint (23) which is nothing but the consistency condition of the equations of motion. The analysis of the constraint presented in the Appendix shows that generically it describes a balance between the matter density and dark energy density. It turns out that in the case

of a macroscopic body, that is in normal conditions, the constraint allows this balance only in such a way that the dilaton practically decouples from the matter and Einstein's GR is restored automatically. Thus our model not only explains why all attempts to discover a scalar force correction to Newtonian gravity were unsuccessful so far but also predicts that in the near future there is no chance to detect such corrections in the astronomical measurements as well as in the specially designed fifth force experiments on intermediate, short (like millimeter) and even ultrashort (a few nanometer) ranges. This prediction is alternative to predictions of other known models.

Formally one can consider the case of a very diluted matter when the matter energy density is of the order of magnitude comparable with the dark energy density, which is the case opposite to the normal conditions. Only in this case the balance dictated by the constraint implies the existence of a non small dilaton coupling to matter, as well as a possibility of other distinctions from Einstein's GR. However these effects cannot be detected in fifth force experiments now and in the near future. One should also note here that in the framework of the present model based on the consideration of point particles, the low density limit, strictly speaking, cannot be satisfactory defined. An example of the appropriate low density limit (CLEP state) was realized using a field theory model in Ref. [32] while conclusions for matter in the normal conditions were very similar to results of the present paper.

Possible cosmological and astrophysical effects when the normal conditions are not satisfied may be very interesting. In particular, taking into account that all dark matter known in the present universe has the macroscopic energy density many orders of magnitude smaller than the energy density of visible macroscopic bodies, we hope that the nature of the dark matter can be understood as a state opposite to the normal conditions studied in the present paper.

VII. APPENDIX. $\zeta(x)$ WHEN THE MATTER IS IN NORMAL CONDITIONS

As we mentioned in Sec. 3, solutions $\zeta = \zeta(\phi, X, \tilde{n})$ of the constraint (23) are generically very complicated functions. Nevertheless let us imagine that we solve the constraint, substitute $\zeta = \zeta(\phi, X, \tilde{n})$ into eqs. (18), (19), (22) and solve them with certain boundary or/and initial conditions. Inserting the obtained solutions for $\phi(x)$ and $X(x)$ back into

$\zeta = \zeta(\phi, X, \tilde{n})$ we will obtain a space-time dependence of the scalar field $\zeta = \zeta(x)$.

Let us analyze possible regimes for the $\zeta(x)$ having in mind its possible numerical values. As we have seen at the end of Sec. 4, in the vacuum $|\zeta^{(\tilde{n}=0)}| \sim b_g$. One can start asking the following question: what is the effect of inserting dust (into a vacuum) on the magnitude of $\zeta(x)$ in comparison with $\zeta^{(\tilde{n}=0)}$? One can think of three possible regimes: $|\zeta(x)|$ may become significantly larger than b_g , may keep the same order of magnitude $|\zeta(x)| \sim b_g$ as it was in the vacuum and may become significantly less than b_g . Consider each of these possibilities.

1. $\zeta(x) \gg b_g$ Let us start from the notion that if formally $\zeta \rightarrow \infty$ then for any particle density $\tilde{n} \neq 0$, the l.h.s. of the constraint (23) approaches zero while the r.h.s. approaches infinity. Therefore a regime where $\zeta \rightarrow \infty$ is impossible.

Consider now the case $\zeta(x) \gg b_g$ with finite ζ . We start from estimations of the order of magnitude of two terms of $V_{eff}(\phi; \zeta)$, Eq. (21), in the vacuum, i.e. $V_{eff}(\phi; \zeta)|_{\zeta=\zeta^{(\tilde{n}=0)}}$. Using Eq. (31) we have

$$\left(\frac{b_g [M^4 e^{-2\alpha\phi/M_p} + V_1]}{(\zeta + b_g)^2} \right)_{vac} \sim \frac{M^4 e^{-2\alpha\phi/M_p} + V_1}{b_g} \quad (36)$$

and

$$\left(\frac{|V_2|}{(\zeta + b_g)^2} \right)_{vac} \sim \frac{|V_2|}{b_g^2} \quad (37)$$

In the presence of dust, in the regime $\zeta(x) \gg b_g$ we have respectively:

$$\left(\frac{b_g [M^4 e^{-2\alpha\phi/M_p} + V_1]}{(\zeta + b_g)^2} \right)_{\tilde{n} \neq 0} \ll \frac{M^4 e^{-2\alpha\phi/M_p} + V_1}{\zeta} \ll \frac{M^4 e^{-2\alpha\phi/M_p} + V_1}{b_g} \quad (38)$$

and

$$\left(\frac{|V_2|}{(\zeta + b_g)^2} \right)_{\tilde{n} \neq 0} \sim \frac{|V_2|}{\zeta^2} \ll \frac{|V_2|}{b_g^2} \quad (39)$$

Therefore generically

$$V_{eff}(\phi; \zeta)|_{\tilde{n} \neq 0} \ll V_{eff}(\phi; \zeta)|_{vac} \quad (40)$$

where we have ignored possible different values of ϕ in the vacuum and inside the matter[53]. Further, proceeding in the same manner with the constraint (23) and using the above estimations it is easily to see that in the regime $\zeta(x) \gg b_g$, the absolute value of the l.h.s. of the constraint (23) is much less than the vacuum energy density. But the r.h.s. of the constraint (23) is of the order of the dust contribution to the energy density (see the last term of Eq. (18) in the regime $\zeta(x) \gg b_g$). Therefore in normal conditions (large \tilde{n}) the constraint (23) does not allow the regime $\zeta(x) \gg b_g$.

2. $|\zeta(x)| \sim b_g$ In this case the l.h.s. of the constraint (23) has the order of the vacuum energy density. Let us start from the assumption that $\zeta(x)$, being $|\zeta(x)| \sim b_g$, is different from the value $\zeta = b_m - 2b_g$. Then the r.h.s. of the constraint (23), being equal to the dust contribution to the pressure (the last term of Eq. (19)), has also the order of magnitude of the dust contribution to the energy density (the last term of Eq. (18)). Therefore in normal conditions (large \tilde{n}) the constraint (23) cannot be satisfied if the value ζ is far from $b_m - 2b_g$. The only way to satisfy the constraint (23) in the regime $|\zeta(x)| \sim b_g$ when the dust is in normal conditions is the equality (33). Consequences of this condition are discussed in Sec. 5.
3. $|\zeta(x)| \ll b_g$ In this case the l.h.s. of the constraint (23) has again the order of the vacuum energy density. But the r.h.s. of the constraint (23) has generically the same order of magnitude as the dust contribution to the energy density (the last term in Eq. (18)). Therefore in normal conditions, the constraint allows the balance (in order of magnitude) between the dark energy density (in the l.h.s. of the constraint) and the r.h.s. of the constraint provided a tuning of the parameters $b_m \approx 2b_g$. Thus the regime $|\zeta(x)| \ll b_g$ is a particular case of the solution (33) if the relation between the parameters b_g and b_m is about $b_m \approx 2b_g$.

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- [48] The more detailed description of the model [31] and its results, including new effects as neutrino dark energy which appear when the fermion density is very low, are presented in Ref. [32].
- [49] Possible nature of the measure fields φ_a have been discussed in Ref. [40]. It is interesting that the idea of T.D. Lee on the possibility of dynamical coordinates[42] may be related to the measure fields φ_a too. Another possibility consists of the use of a totally antisymmetric three index field [40].
- [50] Another way to construct a measure of integration which is a total derivative was recently studied by Comelli in Ref. [43]. Using a vector field (instead of four scalar fields φ_a used in TMT) and proceeding in the second order formalism, it was shown in [43] that it is possible to overcome the cosmological constant problem.
- [51] Note that by the definition (3) the measure Φ is not positive definite. In the Measure Theory the non-positive definite measure is known as "Signed Measure", see for example Ref. [45].
- [52] Note that analogous result has been observed earlier in the model[31],[32] where fermionic matter has been studied instead of the macroscopic (dust) matter in the present model.
- [53] If $V_1 > 0$ then in the late universe $\phi \gg M_p$, $M^4 e^{-2\alpha\phi/M_p} \ll V_1$ and the universe is driven[41] mainly by the cosmological constant $\Lambda = V_1^2/[4(b_g V_1 - V_2)]$.